The layman’s guide to the Magic Cube

Nov. 2017

Preface and expectation

- This presentation is inspired by *Mathologer* and *Nan Ma*'s youtube videos (And you see what a layman I am, I learn stuff from youtube)
- Prior knowledge about…
  - Rubik's cube would be better
  - Group theory would be best, but not necessary

- After this presentation, you should be able to…
  - Solve many weird magic cube by yourselves, within weeks, or hours
- Show me what you got

Intro

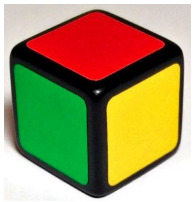
Two types of radical idea

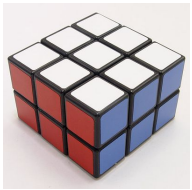
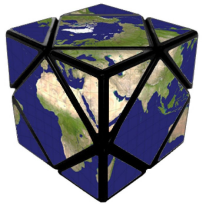
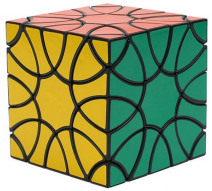
- Magic cube is all about memorizing formulas
- It would take me years to figure out a cube by myself, which makes self-solving boring and pointless
- Formula ruins magic cube

Ladder of difficulty

Complicated ≠ Complex

Which one below is more difficult?

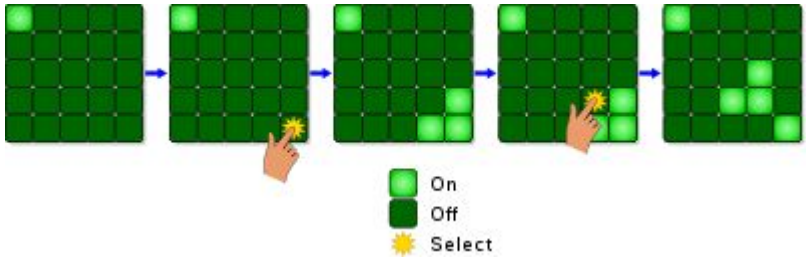


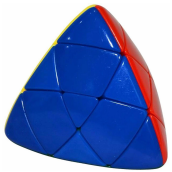
- ???How can we define difficulties for cubes
  - God number?
  - Number of States?
  - Coupling strength?
  - Complexity theory?

Magic cube families

- Commutative puzzle
  - Light-out!
  - Rubik's clock
- Non abelian puzzle
  - Rubik's cubes
  - Some weird shaped cubes
- Bandaging(Jumbling) puzzle ("not-even-a-group puzzle")
  - Square-1 cube
  - Unfortunate twin cube
  - Clover cube



Shape-shifting are not necessarily Bandaging!



Light-out!

- General solution: Matrix Pseudo-Inversion

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix} \overrightarrow{Op.} = \overrightarrow{config.}$$

In[5]:=

```
RowReduce[{{1,1,1,0,1,0,0,0},{1,1,0,1,0,1,0,0},{1,0,1,1,0,0,1,0},{0,1,1,1,0,0,0,1}}, Modulus → 2] //
MatrixForm
```

Out[5]//MatrixForm=

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right)$$

- Never heard about computer? *Chasing light method*
- Nan ma's light out 3D

Tricks for solving non abelian cube

What is “solving a cube” and why it’s hard

- Restoring **states** of a group of objects through definite operations
  - Not “gathering color”
  - Not “gathering color”
  - Not “gathering color”
  - “**States**”: usually means **position and rotation** of various type of blocks

- Operation are not commutative (AB ≠ BA)
- Too many blocks are involved
- Coupling is too strong/long-range

Permutation can be even odder...

- **Even(odd)** permutation is a permutation that realizable through **even(odd)** number of two-exchange.
- 3-cycle is **even**
- 4-cycle is **odd**
- **...**

3-cycle can solve any even permutation

Equivalent object and Symmetry operation

- **EO**

Two objects are said to be equivalent

if you can’t distinguish them after **Tearing Off Stickers**

- **SO**

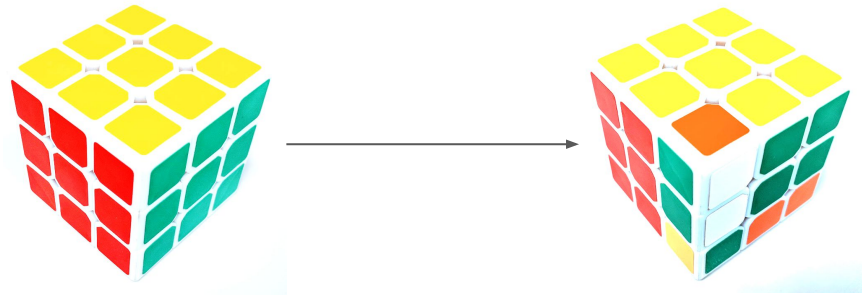
A symmetry operation is such an action

that you wouldn’t notice it if you **Tearing Off Stickers**

Decouple

symmetry operand Ω can be solved, without additional effect, once you can decouple one object from it

- Up to **even parity**



If you can

- Decouple position state: even-cycle among equivalent blocks can be constructed
- Decouple rotational state: spin can be exchanged

Commutator: $ABA^{-1}B^{-1}$

A: Decouple operation (that you invent)
B: Symmetry operation on **Ω**
$A^{-1}(B^{-1})$: Reverse everything you did for A(B)

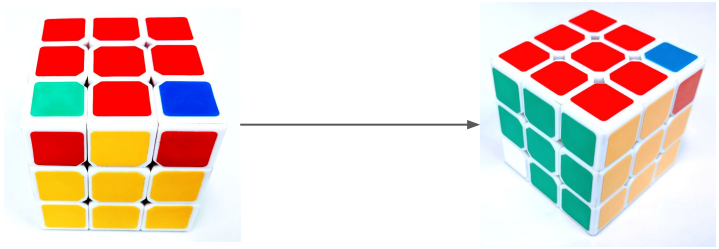
- Commutator measure “how commutative two operations are”
- Commutator always involve **even** number of operations
- An impressive youtube vedio explaining commutator

Similar transformation (Conjugate) MSM^{-1}

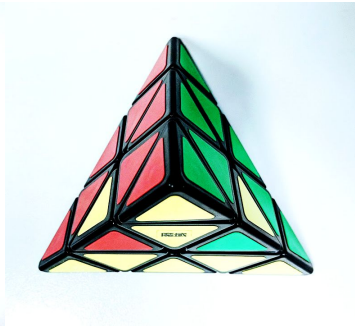
What if you want to solve objects that are not on **Ω** ?

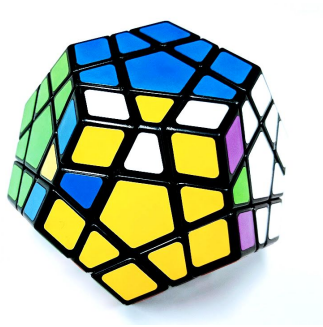
- Find another **Ω'** that involve these objects
- Or…
  - Move those objects into **Ω;**
  - Solve **Ω** by commutator;
  - Move those objects back;

"Operations before and after similar transformation look similar."



Exercise

Challenge:
Solve edge and
corner


Commutator solve everything?

Recall: commutator is **parity free**

Problem doesn't exist if all **generators** are parity free

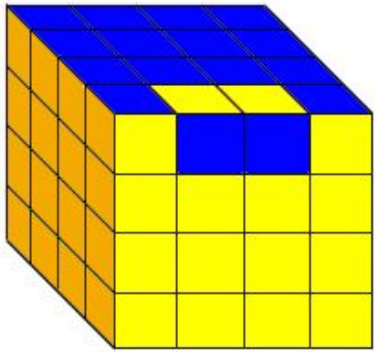
Unfortunately, many cubes (e.g. Rubik's cube) have parity, while it's not a big deal if you know **where parity comes from**.

Catch the parity: where do they come from

333 Rubik’s cube parity?

444 cube: (layman’s nightmare parity):

Square-1 cube parity:



General process of solving a cube

1. Identify features (Class, type of blocks, coupling, symmetry operations, possible sectors)
2. Design a solving order for different types of block (such that less steps are needed)
3. Solve cube up to even parity by using Commutator & Conjugate
4. Jump out of parity(sector)
5. Sometimes inventing “cheating algorithm” could also help

Remaining problems

- Not-even-a-group puzzle: how to jump out of sectors in general?
- Do all twisty puzzle belongs to P problem?
- Can you write a program that generate strategy for any twisty puzzle?
- Are there any other exciting algorithms can help develop general solutions?

Do-not-miss links and reference

http://nan.ma/

http://superliminal.com/cube/cube.htm

http://twistypuzzles.com/

https://www.jaapsch.net/puzzles/

http://oskarvandeventer.nl/index.html

https://youtu.be/B-oePE5gizs

https://www.youtube.com/channel/UC1LJGmvzWdNn_41284Aa2xQ